ETGG 1803
Spring 2008

Lecture 03: Vectors in 3 space

1. Points in 3-space
   a. As we've discussed, a point in 3D space (3-space) is composed of 3 numbers: x, y, z
   b. I'll usually call this a Vector3
   c. Recall that if we *visualize* a 3D vector, it is important whether this is a right-handed or left-handed system.
   d. Mathematically, though, that doesn't matter.
   e. What would happen if we define a model in a left-handed coordinate system and display it in a right-handed system?
2. Vector3 in 3-space
   a. Even though we can (and will) represent points using our Vector3 class, they have another, more general, meaning…
   b. It is a **direction**
   c. Example:

$$\vec{C} = \begin{bmatrix} -3 \\ -2 \\ 4 \end{bmatrix}$$

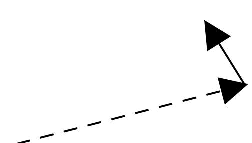
   d. This is a direction from some point in space then
      i. 3 units along the –X direction
      ii. 2 units along the –Y direction
      iii. 4 units along the +Z direction.
   e. Note that this doesn't say anything about absolute positions in 3-space.
      i. To have an absolute position we need:
         1. A starting point.
         2. A set of three coordinate axes.
      ii. If we have the direction vector C, it's the same vector here… (and here…)
   f. A vector also implicitly describes a distance travelled.
      i. How do we figure this out?
         1. If we've moved 3 units in the –X, 2 units in the –Y, and 4 units in +Z, how far away are we from our starting point?
         2. Picture…
         3. How would we do it in 2D…Pythagorean!
         4. It actually works in any dimension:
      ii. Length of a Vector3:
         1. $Length(\vec{V}) = \|\vec{V}\| = \sqrt{V_x^2 + V_y^2 + V_z^2}$
   g. Vector vs. Scalars
      i. So, a Vector describes two things:
         1. A direction in 3-space
         2. A distance travelled in 3-space (magnitude or length)
      ii. Scalar:
         1. A scalar is a fancy name for a single-value (instead of a Vector's multiple values).

iii. An example:
1. When someone says I was going 80mph, that's a scalar.
   a. It doesn't tell you anything about the direction they were travelling.
2. When someone says I was going 80mph when going from Portsmouth to Lucasville, that's a velocity vector.
   a. It has a magnitude (80mph)
   b. And a direction (north)

h. Vector3 and Points
   i. A Displacement from a point
      1. One way we can use Vector3 is to specify a movement (**translation**) from one point to another.
      2. Let's say we're at the point [0.5, 3.2, -1.9] and we go in the direction specified by C.
      3. Now we're at: [-2.5, 1.2, 2.1]
      4. We'll see how to calculate a Vector3 between two points shortly.
   ii. Points as a Vector3
      1. When I say a Point P, I really mean start at the origin, go in the direction specified by the *Vector* P.
      2. So, a Point is just a Vector3 that's an offset from the origin!

i. A few special terms:
   i. **zero vector**: Just a vector with all the elements 0.0. If we think of this as an offset, it just means don't move at all.
   ii. **unit-length** (or **normal**) vector. A vector with length 1.0. We'll see that there are some special things you can do with normal vectors later (and how to calculate them).

3. Properties of Vectors
   a. For each property, we can look at the property in 3 ways:
      i. Symbolically: $\vec{a} = \vec{b} + \vec{c}$
      ii. Numerically: $\begin{bmatrix} 4-1 \\ 3+2 \\ 0+9 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ 9 \end{bmatrix}$
      iii. Graphically:



   b. Property1: **The negative of a Vector**
      i. Symbolically: **-**$\vec{a}$
      ii. Numerically: negate each component (multiply them by -1)
      iii. Graphically: along the same "line", but in the opposite direction.
   c. Property2: **Vector Length**
      i. Symbolically: $\|\vec{a}\|$
      ii. Numerically: Use the 3d Pythagorean theorem.
      iii. Graphically: The length of the vector.

d. Property3: **Normalized Vectors**

i. Symbolically: if $\vec{a}$ is a (possibly) non-normal length vector, $\hat{a}$ is the normalized version of this.

1. $\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$
2. They both have the same direction
3. Just (possibly) different lengths.
4. All components of a normalized vector are less than or equal to 1.0

ii. Numerically:

1. Calculate the Length, L, of a.
2. Divide each component of a by L.

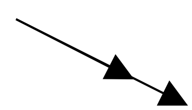
iii. Graphically:

e. Property4: **Multiplying a Vector by a scalar.**

i. Symbolically: c $\vec{a}$

ii. Numerically: $1.5\begin{bmatrix} 2.0 \\ -1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 1.5*2.0 \\ 1.5*-1.0 \\ 1.5*0.0 \end{bmatrix} = \begin{bmatrix} 3.0 \\ -1.5 \\ 0.0 \end{bmatrix}$

iii. Graphically:



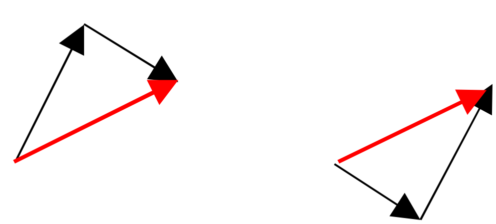
f. Property5: **Vector Addition**

i. Symbolically: $\vec{c} = \vec{a} + \vec{b}$

ii. Numerically: $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2 \\ 3+(-1) \\ -1+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

iii. Graphically:

1. Take the base of one arrow and place it at the tip of the other.



2. Note that both are just directions, so the result is just another direction.
3. If we think of the first vector as a point and the second as a direction, the result is another point.

iv. Other:
   1. Vector addition is commutative and associative. a + b is the same as b + a, and in a + b + c, it doesn't matter what order you evaluate the operators.
   2. If you add the zero vector to any vector, you get that vector as a result.

g. Property6: **Vector Subtraction**
   i. Symbolically: $\vec{c} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$
   ii. So…it's just a vector addition with negative b.
   iii. Numerically:
   iv. Graphically:
   v. Other:
      1. If we have two points a and b,
         a. the direction from a **<u>to</u>** b is b – a
         b. the direction from b **<u>to</u>** a is a – b
         c. We'll use this a lot.
         d. We can calculate the distance between the two points by computing this vector and then calculating the length of it.
         e. Often the normalized direction from a to b is very useful (it'll be one thing we add in the raytracer very early).