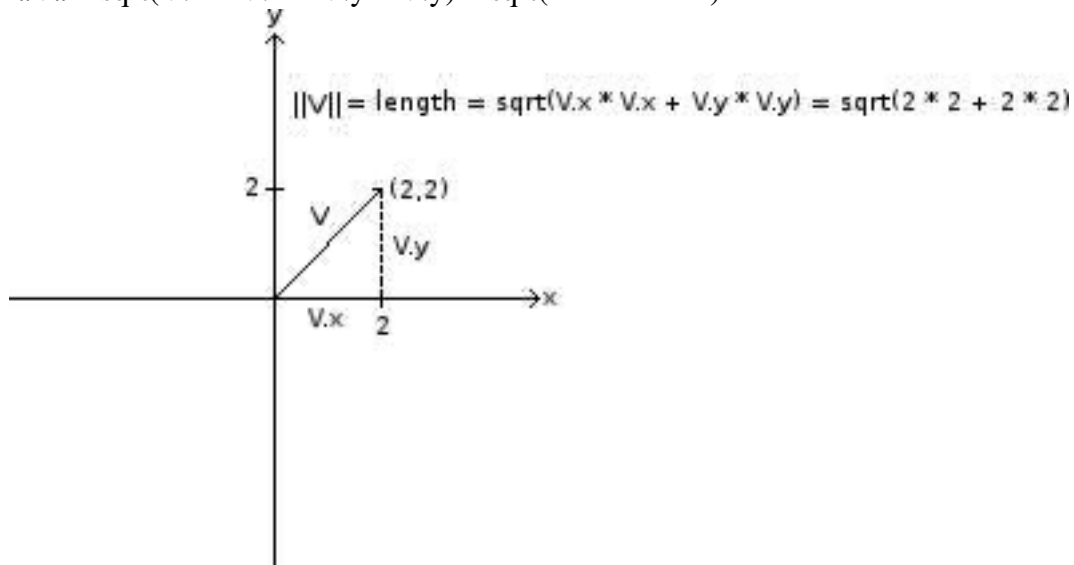


Vector

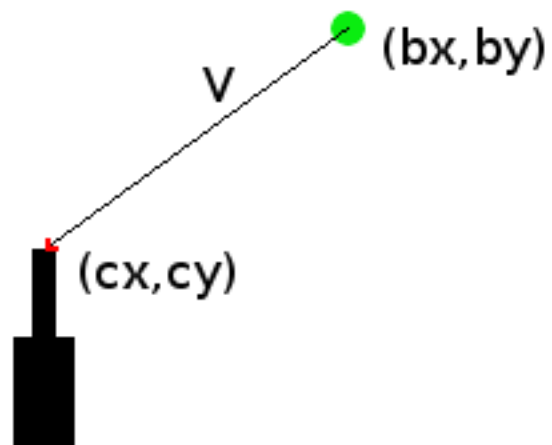
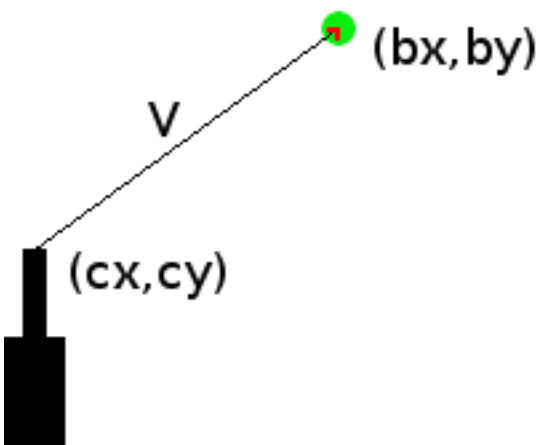
- Like a point.
- Represents magnitude (length) and direction instead of position.
  - Forces: gravity, wind, etc.
- Length is found using the Pythagorean Theorem.
  - $V = (2, 2)$
  - $\|V\| = \text{sqrt}(V.x * V.x + V.y * V.y) = \text{sqrt}(2 * 2 + 2 * 2)$



- Find the direction in which to fire a cannon ball.
  - Use the difference between two points to create a vector.

$$V = (bx - cx, by - cy)$$

$$V = (cx - bx, cy - by)$$

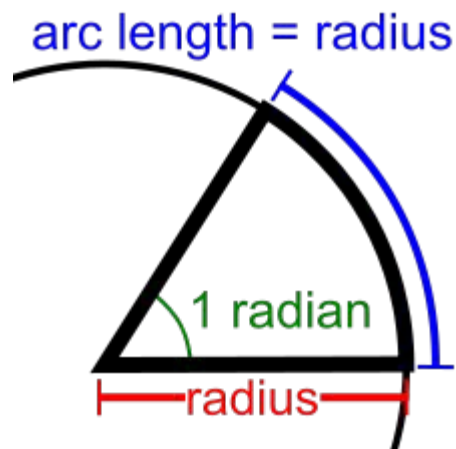


- The vector's length will be the distance between the two points. Normalize to make its length equal 1.

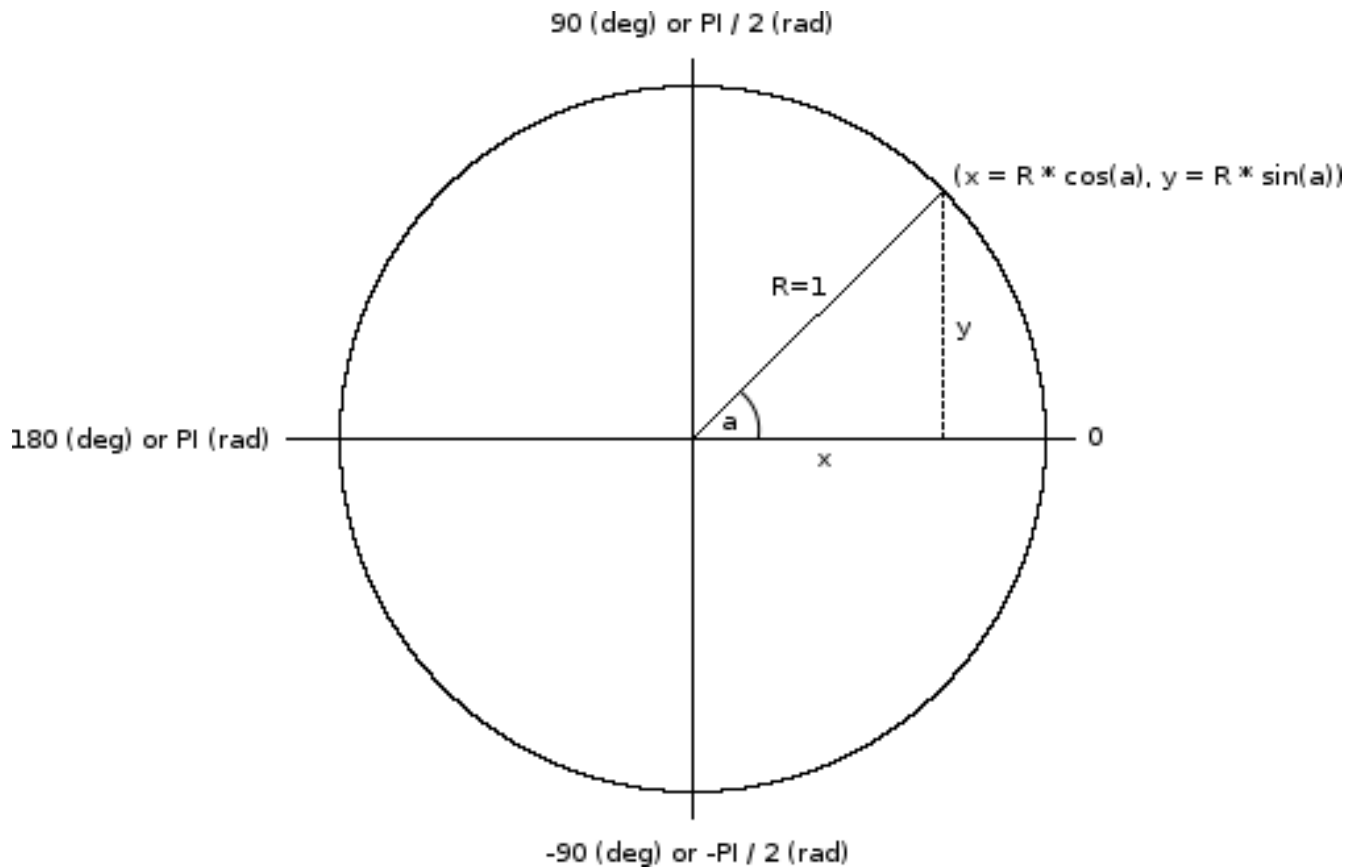
- Normalizing a Vector
  - Makes vector have a length of 1 without changing its direction.
  - Allows us to easily customize the vector's length.
    - Ex: To represent the speed of a bullet.
  - Process:
    - Find vector's length.
    - Divide vector's components (x, y, z, ...) by its length.
      - $V = (2, 2)$
      - $\|V\| = \sqrt{2 * 2 + 2 * 2} = \sqrt{8}$
      - $V \text{ (normalized)} = (V.x / \|V\|, V.y / \|V\|) = (2 / \sqrt{8}, 2 / \sqrt{8}) = (0.7071, 0.7071)$

### Rotation

- Trigonometric functions
  - $\sin = \text{opposite} / \text{hypotenuse}$
  - $\cos = \text{adjacent} / \text{hypotenuse}$
  - $\tan = \text{opposite} / \text{adjacent}$
  - Function of an angle
  - SOH-CAH-TOA
- Radian
  - Equal to the angle when arc length equals radius



- Python's trig functions use radians
  - `math.sin`
  - `math.cos`
  - `math.tan`
- $1 \text{ radian} = 180 / \text{PI} \text{ (degrees)}$
- $90 \text{ (deg)} = 90 * \text{PI} / 180 \text{ (rad)}$
- $x = \text{length} * \cos(\text{angle})$
- $y = \text{length} * \sin(\text{angle})$



- Inverse trigonometric functions
  - Cannon example: using direction vector, find angle to rotate the sprite by.
  - arcsin:  $y = \sin(x)$ ,  $\arcsin(y) = x$
  - arccos
  - arctan
  - Used to find angle
    - $V = (2, 2)$
    - angle = unknown
    - $\sin(\text{angle}) = \text{opposite} / \text{hypotenuse} = V.y / \|V\| = 2 / \sqrt{8}$
    - $\arcsin(\sin(\text{angle})) = \arcsin(2 / \sqrt{8})$
    - $\text{angle} = \arcsin(2 / \sqrt{8}) = 45 \text{ (deg) or } \text{PI} / 4 \text{ (rad)}$
  - $(2, 2)$  works,  $(-2, 2)$  does not.
    - $V = (-2, 2)$
    - angle = unknown
    - $\sin(\text{angle}) = \text{opposite} / \text{hypotenuse} = V.y / \|V\| = 2 / \sqrt{8}$
    - $\arcsin(\sin(\text{angle})) = \arcsin(2 / \sqrt{8})$
    - $\text{angle} = \arcsin(2 / \sqrt{8}) = 45 \text{ (deg) or } \text{PI} / 4 \text{ (rad)}$ 
      - This is the same answer! It should be 135 (deg).
  - $\sin(45)$  has the same value as  $\sin(135)$ .

## ETGG1801

### Game Programming Foundations I

Fall 2008

- Quadrants 1 and 4 (when  $V.x$  is positive) work; 2 and 3 (when  $V.x$  is negative) do not.
- Apply some logic to fix this:
  - if  $x < 0$ :
    - $\text{angle} = 180 - \arcsin(y / \text{length})$
  - else:
    - $\text{angle} = \arcsin(y / \text{length})$
- Python's `math.atan2` function does this for us!